# Enhanced Simulated Annealing Through Linear Programming Preprocessing

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## Abstract

Simulated annealing (SA) is a heuristic technique which has been successfully applied to a wide range of optimisation problems. Despite its wide applicability, it does have the drawback that the final solution could be suboptimal. Moreover, in problems with enormous solution spaces, the search for good solutions can be long. In contrast, techniques such as branch and bound provide optimal solutions by systematically pruning the search space with the help of information about the problem. In this paper we describe a scheme for combining a relaxed linear program (LP) of a 0-1 optimisation problem with an SA search. This improves the performance of the SA by pruning the search space as it proceeds. Further pruning is achieved by investigating the structural properties of the problem. Also, the lower bound from the relaxed LP provides a measure of the quality of the SA solution. We are not aware of this approach being used with simulated annealing. We illustrate the technique by applying it to the set partitioning problem (SPP) and give some performance results.

# 1 Introduction

Simulated annealing (SA) is a *Monte Carlo* method, based on the analogy between the aggregation of many particles in a physical system as it is cooled and the optimisation of large combinatorial problem (see, for example, van Laarhoven & Aarts [1987]). In the physical system, annealing is the process of heating a solid until it melts, followed by a cooling operation until the substance crystallises. Fast cooling leads to a process called *quenching* in which the crystal which is formed has a metastable amorphous structure (or imperfect lattice), with a very large internal energy. Slow cooling allows the system to reach its equilibrium (lowest energy level) at each decremental temperature. Hence it is possible to reach the zero energy level or ground state. In the optimisation perspective, search space can be viewed as a physical system in which elements or variables correspond to particles. The atomic bonding force is equivalent to the set of constraints imposed on the problem. The energy of the system is modelled as the penalty cost, so that when the system reaches the ground state, the process arrives at the global optimum configuration.

Starting off at maximum temperature, the metal liquid is cooled. At each temperature, thermodynamic equilibrium is established. The probability of a given state, $\boldsymbol{E}$, having an energy $E$ is given by the Boltzmann distribution shown below:

$$Pr\{\boldsymbol{E} = E\} = \frac{1}{Z(T)} \times e^{\frac{-E}{kT}}$$

Where
$k$ = Boltzmann Constant
$T$ = Temperature
$Z(T)$ = Normalisation Function
$e^{\frac{-E}{kT}}$ = Boltzmann Factor

SA is a probabilistic process which is similar to the well known local search descent algorithms (Aarts & Korst [1989]). In such algorithms, the local *neighbourhood* of a current solution is recursively searched for improvements (known as *down hill* moves)

until none are possible. In contrast to descent algorithms, however, SA allows *up hill* movements that lead to disimprovements in the solution quality in order to escape from local minima. In SA, all down hill movements are accepted because they decrease the system energy. At higher temperatures up hill movements are accepted readily. As the temperature decreases, the chance of going up hill is decreased; up hill movement becomes impossible when the mass is frozen.

Given a neighbourhood structure, SA attempts to transform the current solution into one in its neighbourhood. This mechanism is mathematically described as a *Markovian transformation* where the current structure depends only on the previous one (see van Laarhoven & Aarts [1987]). To simulate the evolution of the SA algorithm, a *starting temperature* is computed so that almost all transitions from all neighbourhood configurations are accepted. The algorithm iterates by perturbing the current configuration and measuring the change in cost. If the change in cost is negative, the new configuration is automatically accepted. Otherwise the probability of accepting the increase is computed by evaluating the Boltzmann's factor. If the probability is greater than a random number in the interval [0,1], the new configuration is accepted, otherwise, it is rejected. At each temperature, the process is repeated an appropriate number of times, called a *Markov chain length.* The temperature is then decreased according to one of a number of *cooling schedules* (see van Laarhoven & Aarts [1987]). The process is repeated until the system is frozen or no change in cost between a number of consecutive Markov chains is detected.

SA has the advantage of searching for a solution without prior knowledge of the problem. It has been used for solving combinatorial, integer, linear and non-linear problems with very little change to the core algorithm. There are, however, two major disadvantages of SA. First, because all practical cooling schedules are not infinitely long, it gives no guarantee that the solution returned is optimal. Worse still, it is not possible to measure how far the solution is from optimal. Secondly, in problems that have

an enormous search space, SA takes no account of problem structure, and thus may consider any point in the search space to be a likely candidate for the solution. The consequence of this is that the search can consume huge amounts of computer time. However, if a reliable lower bound can be determined through some other method, then it is possible to measure the quality of the SA solution. The lower bound can be used to prune sections of the search space, thereby improving the performance of SA. Further, if the problem has a particular structure it may be possible to further reduce the search space by incorporating that knowledge into the core algorithm. In Sections 3 and 5 we introduce such a technique.

## 2 Using SA to Solve 0-1 Integer Programs

Many combinatorial optimisation problems can be formulated as the minimisation of an objective function $C(x)$ where the vector $x = X_j, j = 1, \ldots, N$, consists of integers which can only take the values zero or one. The minimisation is usually subject to a number of constraints which link the value of the free variables so that they obey certain limitations. In the linear system, both the objective function and the constraints are linear, and take the following form:

$$\begin{aligned} min \quad & C(x) = \sum_{j=0}^{n} C_j X_j \\ \text{subject to :} \quad & Ax \leq b \\ & X_j \in \{0, 1\} \end{aligned}$$

Problems of the type described above can often have enormous solution spaces. SA, as described above, has the ability to traverse such spaces in search of global minima. In the context of the 0-1 formulation, the search can proceed by randomly toggling the values of the binary variables, whilst using the Boltzmann distribution to determine whether up hill moves are accepted.

However, the basic SA algorithm described in the introduction takes no account of constraints when it performs the Markovian tranformation. There are two main techniques that can be used to incorporate these constraints. First, they can be enforced when each local move is made, and thus only feasible moves are generated. This approach is described in Connolly [1992], and is very effective when feasibility can be restored easily. It has the advantage that all partial solutions are feasible, and thus the search can stop at any time with a feasible solution. However, it has the disadvantage that it is difficult to generate moves when the problem is tightly constrained. In fact, finding an initial feasible solution may be very hard. The second approach is to relax the constraints and incorporate them into the objective function by using a penalty function. Thus, constraints which are violated force the cost to rise. When the cost is minimised, the original objective is minimised and the constraints are satisfied. This approach has the advantage that the move generator can produce any change without concern for the constraints. Unlike the previous scheme, the search can proceed through infeasible regions in the hope that a feasible, and lower cost, region lies beyond. However, it has the disadvantage that the new objective function is highly non-linear even if the original problem was linear. Thus, it is possible for the search to stop in a local minimum, which may not even be inside the feasible region. Regardless of which generation strategy is used, the basic form of the algorithm is as follows:

Generate initial configuration called $x$
Compute cost of initial configuration $C(x)$
Compute initial temperature $T_0$
Set $T = T_0$
  While not frozen and $C(x)$ non optimal do begin
    Repeat markov-chain-length times begin
      $i$ =random$(0, n-1)$
      Set$x' = x \oplus 2^i$
      If using first generation scheme,
      restore feasibility of $x'$
      $\triangle C = C(x) - C(x')$
      If $(\triangle C \leq 0)$ or $e^{-\triangle C/T} > unif\_rand(0,1)$ then set $x = x'$
    end
  $T = cool(T)$
  end

where:

$a \oplus b$ returns the exclusive-or of $a$ and $b$
random $(a, b)$ returns a random uniformly distributed integer between $a$ and $b$
*unif_rand*$(a, b)$ returns a random uniformly distributed real between $a$ and $b$
*cool* $(T)$ returns a new value for the temperature after applying a cooling operation to $T$. In the experiments reported in this paper we use a reheating scheme in which the temperature is reduced and then raised to escape from local minima.

As described in the introduction, SA takes no account of the problem structure and simply randomly makes moves in the search space. There is nothing to prevent the search from cycling or generating moves into regions which cannot be in the final solution. In the next section we describe a technique which allows the algorithm to reduce the size of the search by setting certain values in the $x$ vector to zero. In this way they can be removed from the search.

# 3 Preprocessing 0-1 IPs Using LP Lower Bounds

Consider the 0-1 integer programming problem,

$$P_{0-1}: \quad Z_{0-1} = \quad min \quad Cx \tag{1}$$

$$\text{subject to}: \quad Ax \leq b \tag{2}$$

$$X_j \in \{0, 1\} \tag{3}$$

Here, $C$ is an $n$–vector, $A$ is an $(mXn)$ matrix and $b$ is an $m$–vector. The problem $P_{0-1}$ has an optimal solution value, $Z_{0-1}$.

A relaxation of $P_{0-1}$ is a problem in which some of the difficult constraints have been either ignored or simplified. In particular, the $LP$ relaxation of $P_{0-1}$ is one wherein constraint (3) is simplified.

Consider the problem $LP_{0-1}$:

$$LP_{0-1}: \quad Z_{LP} = \quad min \quad Cx \tag{4}$$

$$\text{subject to}: \quad Ax \leq b \tag{5}$$

$$x \leq 1 \tag{6}$$

$$x \geq 0 \tag{7}$$

$LP_{0-1}$ is an $LP$ relaxation of $P_{0-1}$. It follows that $Z_{LP} \leq Z_{0-1}$. In other words, the optimal solution to $LP_{0-1}$ forms a lower bound to the solution of $P_{0-1}$ This lower bound is quite useful in several respects. First, it can be used to measure the quality of the upper bounds. Let $Z_{UB}$ be the upper bound obtained through the SA heuristic. The optimal solution to the $P_{0-1}$ is such that $Z_{LP} \leq Z_{0-1} \leq Z_{UB}$. The *gap* between $Z_{LP}$ and $Z_{UB}$ can, therefore, be used to gauge the effectiveness of the upper bounding algorithm. Second, the lower bound can be used in conjunction with the upper bound to perform preprocessing operations. Given a formulation of a problem, preprocessing refers to operations that improve or simplify it by tightening bounds, fixing values of variables, and so on.

Consider the problem $LP_{0-1}$ and let $x^{LP}$ be the optimal solution vector. Let $x_B^{LP} \subset x^{LP}$ be the set of basic variables and let $x_J^{LP} \subset x^{LP}$ be the set of non basic variables in the $LP$ optimal solution $x^{LP}$. For simplicity, let us assume that the current optimal basic feasible solution is non-degenerate (that is, for the $m$ constraint $LP$, the current optimal solution has $m$ variables assuming positive values). For any nonbasic variable $X_k \in x_J^{LP}$, we can define the reduced cost, $C_k'$, as the amount by which the objective function coefficient of $X_k$ must be improved (decreased) before $X_k$ could assume a positive value in the optimal solution. Another interpretation of $C_k'$, the reduced cost of a nonbasic variable $X_k \in x_J^{LP}$, is that it is the amount by which the value of $Z_{LP}$ will change (increase, in the case of a minimisation problem) if we increase the value of $X_k$ by 1 from its value of 0 (while all the other nonbasic variables remain equal to

zero), thereby introducing it into the basis. By definition, the reduced cost of basic variables is 0.

In 0-1 integer programming problems, the reduced cost information provides us with a measure to evaluate an increase by 1 (the upper bound) in the activity level of nonbasic variables from their value of 0 in $x_{LP}$. Thus, the introduction of a nonbasic variable, $X_k \in x_J^{LP}$ in the basis has the effect of an increase in the lower bound from $Z_{LP}$ to $Z_{LP} + C'_k$. We can use this information to eliminate variables from the problems and hence, reduce its size.

For all $X_k \in x_J^{LP}$, if $Z_{LP} + C'_k > Z_{UB}, X_k$ can never be a part of any optimal solution to $P_{0-1}$ and hence, can be eliminated from the problem. This is because we know that the optimal solution to $P_{0-1}$ is such that $Z_{LP} \leq Z_{0-1} \leq Z_{UB}$.

In this manner, all nonbasic variables in $x_{LP}$ can be considered in turn as candidates for elimination. This process often results in significant reductions in problem size. We will illustrate this in Section 6.

# 4 The Set Partitioning Problem

Let $M = \{1, \ldots, m\}$ and $N = \{1, \ldots, n\}$ denote the set of rows and columns, respectively, in an $(mXn)$ matrix, $A$. Further, assume that the elements of the matrix $A$ take values of either 0 or 1. Let $a_{ij}$ denote the $(i,j)$th element of $A$. Associated with each column $j$, of the matrix $A$ is a cost $C_j$.

Then, the set partitioning problem (SPP) on matrix $A$ can be formulated as follows:

$$P_{SPP}: \quad Z_{SPP} = \quad min \quad \sum_{j=1}^{n} C_j X_j \tag{8}$$

$$\text{subject to}: \quad \sum_{j=1}^{n} a_{ij} X_j = 1, \quad \forall i \in M \tag{9}$$

$$X_j \in \{0, 1\} \tag{10}$$

A well known application of the SPP is the airline crew scheduling problem (CSP). The goal is to minimise crew costs while satisfying constraints on work/duty content. Given an airline's flight schedule (arrivals and departures at all destinations, including aircraft types), *pairings* are initially formed. Each pairing is a sequence of flight 'legs' which begin and end at a base location, known as *home-base*. Pairings are governed by union rules, aviation rules and operational rules (length of stay away from home-base), and so on. Each pairing also has a cost associated with it. Given an exhaustive set of legal pairings, each with an associated cost, the problem of finding the best set of pairings such that each flight leg is covered once is the SPP, given by (8), (9), and (10).

Given a solution vector $x$, a flight leg is said to be *overcovered* if more than one pairing covers it; it is said to be *undercovered* if it is not covered by any pairing; it is exactly covered if it is covered by exactly one pairing.

Each row of the matrix $A = [a_{ij}]$ represents a flight leg and the 0-1 variables $X_j$ indicate which of the pairings $j (j = 1, ....n)$ are selected in the optimal solution. The matrix $A$ is constructed such that:

$$a_{ij} = \begin{cases} 0, & \text{if flight leg } i \text{ is included in pairing } j \\ 1, & \text{otherwise} \end{cases}$$

CSPs are normally very large problems. For example, for problems involving just 500 flight legs, in excess of a million feasible pairings could easily result. This then results in a 0-1 integer programming problem with more than one million variables and 500 constraints.

Operating costs for the flying crew of some major airlines are in excess of \$1.3 billion each year [Anbil *et al*, 1991]. Moreover, personnel costs are often the second largest item, after fuel, of the operating costs for most airlines. Hence, even small improvements in existing operations can result in huge savings. Thus, good solution methods for these large and difficult 0-1 problems are essential.

Most algorithms for the CSP consist of two steps. The first is the generation of a set of feasible pairings and the second is the solution of the resulting SPP. In the second stage, the SPP can either be solved exactly or by heuristic methods. In this paper, we develop an approximate algorithm for SPP using SA, while constantly attempting to reduce the problem size.

Like the CSP, most other applications of the SPP also arise in the transportation area: scheduling of trucks (Balinski & Quandt [1964]; Cullen, Jarvis & Ratliff [1981]), scheduling of ships (Brown, Graves & Ronen [1987]; Fisher & Rosenwein [1989]), scheduling of airline fleets (Levin [1969]), airline crew scheduling (McCloskey & Hanssmam [1957]; Marsten, Muller & Killion [1979]; Marsten & Shepardson [1981]). Detailed surveys on the SPP, including its applications, can be found in Garfinkel & Nemhauser [1969, 1972], Balas & Padberg [1974, 1976] and in Christofides, *et al* [1979].

Specific algorithms for solving the SPP have been proposed by Balas & Padberg [1975], Garfinkel & Nemhauser [1969], Pierce [1968], Marsten [1974], Nemhauser, *et al* [1974], Pierce & Lasky [1973]. The most effective of these, for a wide class of problems, appears to be the one developed by Marsten [1974].

Most algorithms relax the binary condition on $X_j$, in $P_{SPP}$, thereby obtaining the LP relaxation problems $P_{LP-SPP}$, with the constraints $0 \leq X_j \leq 1$ instead of constraint (10). The non-integral $X_j$'s are resolved through a branch-and-bround tree-search. In initial implementations of branch and bound algorithms, the choice of branch variables and constraints were not sophisticated: The solution space is searched by setting variable values to 0 and 1 alternatively, thereby generating sub-problems and partial solutions. However, a more sophisticated branching scheme was applied by Marsten [1974], in which branching decisions are based on groups of variables, through which the status of individual variables is determined automatically. This approach works particularly well for SPPs with sparse matrices (like that in the CSP).

However, in all of these algorithms, the solution of many LPs is the critical factor affecting computational efficiency of the resulting algorithm. This is because the LP relaxation of $P_{SPP}$, being highly degenerate in most instances, is difficult to solve. Hence, although solutions to $P_{LP-SPP}$ are normally integral and although $Z_{LP-SPP}$ is, in practice, a tight lower bound on $Z_{SPP}$, the branch and bound algorithm that uses the LP relaxation is often computationally expensive. In a recent paper, Hoffman & Padberg [1992] solve CSPs using what they call a *branch and cut* algorithm. This has proved quite effective even for large problems.

In this paper we demonstrate the use of SA to solve the set partitioning problem. The method will be discussed in the following section.

# 5 Using SA to Solve the Set Partitioning Problem

In Section 2 we described a general SA algorithm capable of solving general 0-1 problems. In this section we discuss some of the features of the solution which are particular to the set partitioning problem. These are the internal data structures, the structural properties which can be exploited, the cost function structure adopted, the choice of the move operator and the cooling function.

Our choice of problem (the SPP) is not intended to demonstrate an algorithm which is more effective at solving the SPP than the specific algorithms in the literature that have been reviewed in Section 4. Rather, it provides a test problem which is widely recognized as difficult. Further, our SA algorithm makes use of the technique of lower bounding of the objective function as used in the branch and bound techniques used for solving SPPs. Thus we demonstrate that this scheme has value in simulated annealing.

## 5.1 Data Structures

A sparse matrix representation is employed to denote the characteristic of the data structure. Each pairing is uniquely defined by its own features such as cost, number of flights, LP reduced cost, etc. Each pairing also points to a chain of non-zero flight leg elements which are the flight legs that it covers. Pairings are packed into a one dimensional array for the ease of indexing.

A hash table is generated at the beginning of the run. It uses binary representations of flights and pairings to make the process of swapping columns easier.

A solution array is created to hold the column indices which are currently in the solution *i.e.*, $j$ such that $X_j = 1$. Also we maintain a flight counter to keep track of the current number of flight legs in the solution, and the number of flight legs that cover each row after problem reduction is completed. The latter helps the second pruning scheme using the structural properties of the SPP.

## 5.2 Structural Properties

Apart from the problem reduction scheme (mentioned in Section 3) using the LP relaxation, the second pruning scheme can be derived based on the SPP structural property. Let $J$ be the set of pairings associated with nonzero entries in row $i$. If one of the pairings, $k \in J$ is known to be in the solution, then every pairing $l \in J, l \neq k$ must be zero in all feasible zero-one solutions.

This structural property can be incorporated into the problem by maintaining the number of flight legs remaining for each row, $i$. If one of the flight counters gets down to 1, the column containing that flight leg must be in the solution. We call this column an *indispensable* column. Let us assume that column $j$ is indispensable. Assume further that column $j$ also covers flight leg $l$. Clearly, since $j$ must be in the optimal solution,

flight $l$ is also automatically covered. Furthermore, all other columns that cover $l$ (other than $j$) can be removed from the problem as they become *superfluous*. This process is carried out till no other superfluous columns can be identified.

This results in a very low number of columns at the end of the SA for many of the test problems we used. The number of columns remaining is negligible even to perform an exhaustive search to prove the global optimality.

## 5.3 Cost Function Structure

The relaxation of the constraint (9) results in a second type of cost, called *penalty cost*, beside the actual operating cost, $C_j$, which acts on the rows of the problem. The latter operates on the columns. Let the *set cost* be the sum of all the row penalty costs. The penalty cost is positive when the SA solution violates some of the row sum constraints. Clearly, when an intermediate SA solution is such that, all rows are covered exactly once, the penalty cost for all rows is zero. The set cost for this solution is also zero. If a row is under covered or over covered, we impose a penalty depending on the amount of violation. The profile of the penalty cost curve we have used is shown in Figure 1.

The profile of the penalty cost has to be chosen carefully. If the cost for deviation from a row sum of 1 is too high, then it will be more difficult for a new configuration to be generated. However, if the penalty for deviation is too low, infeasible solutions will be readily admitted and it is likely that the final solution will be infeasible.

Moreover, to prevent excessive over covering of a flight, a large penalty cost is assigned to rows that are covered by more than Maxcount ($M_c$) pairings.

Although the introduction of such a penalty (set) cost makes the original linear cost function nonlinear, it removes the constraints in the problem.

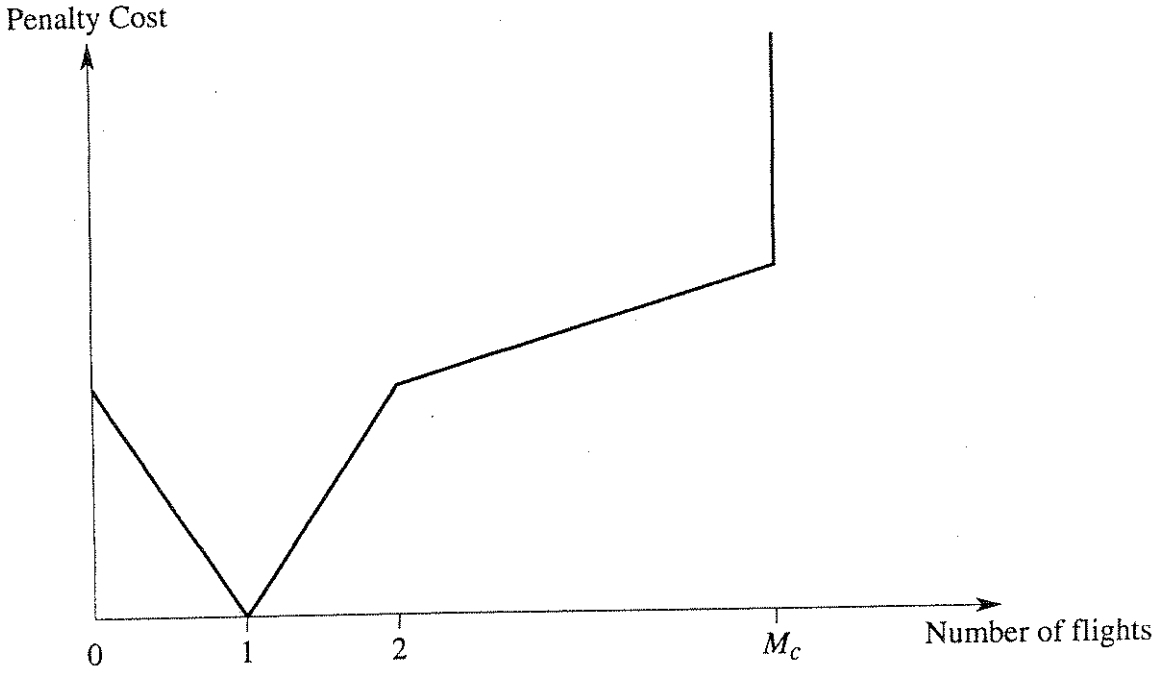


**Figure 1: Penalty cost profile for the SPP**

## 5.4 The Move and Swap Operators

Given an initial configuration, a transition is performed within the current configuration's neighbourhood. This is achieved by randomly choosing a variable. We take it out of the solution if it is currently in it or insert it in the solution if it is not. The change in cost is then calculated. If it is accepted by the SA rules, the configuration is updated. Otherwise the old configuration is retained.

With a finite cooling schedule, the SA algorithm drives the system to a local optimum solution. The gap between this local optimum and the global optimum depends on the number of permutations performed at each temperature (or the Markov chain length) and the cooling schedule. Faster cooling and short Markov chains lead to poorer final solutions. If the system is trapped in local minima at low temperatures, the chance of escaping from it is difficult, unless the neighbourhood configuration is extended. This

is achieved by either increasing the system energy (described in the next Section) or by a more sophisticated move.

We use a sophisticated K–R opt method for swapping pairings (Fisher & Kedia [1990]). Under this method, a group of pairings, of size K which are currently in the solution is randomly chosen. The union of the flights that are covered by this group is computed. The set of flights covered by the chosen group of pairings is denoted as 'UNISET'. An attempt is made to swap this group with another group of pairings of size R (not necessarily having the same number of pairings) which are not in the current configuration. The new group is chosen such that no flight is covered more than once. Moreover, the union of their flight legs is the same as UNISET.

In the case where R is less than K in the K–R opt procedure, a move operator is also performed in conjunction with these swaps in order to prevent the progressive lack of pairings in the solution as the SA algorithm proceeds.

## 5.5 The Cooling Schedule

A cooling schedule consists of: (i) the starting temperature. (ii) the cooling factor. (iii) the length of the Markov chain. (iv) the process termination condition.

We use a starting temperature based on White [1984], who proposed that the starting temperature should be one standard deviation around $\bar{C}$, the mean cost, based on his configuration density function. We use a cooling factor of 0.9. This is based on Kirkpatrick [1982, 1983] who propose that the temperature decrement rule of should be controlled by a constant cooling factor which is small but close to 1. This rule was used by, for example, Johnson, *et al* [1986], Bonomi and Lutton [1984a, 1984b, 1986], Leong & Liu [1985], Morgenstern & Sharpiro [1986], Sechen & Sangiovanni-Vincentelli [1985]. For simplicity, we used a constant Markov chain length for all the problems we tested our SA on.

The difference between our cooling schedule and those used eariler in the literature is the use of a *temperature reset* procedure which increases the temperature if we detect that the system is trapped in local minima. We say a local minimum is reached if two consecutive Markov chains provide no change in the solution (either uphill or downhill). Furthermore, we say a local minimum is reached if there is a constant fluctuation around a point (present in cases of cycling). At such points, we reset the system temperature according to:

$$T_{new} = P \times C(x^b) + Q$$

where, $T_{new}$ = new reset temperature when a local minimum is detected.
$x^b$ = the current best solution
$C(x^b)$ = the cost of the current best solution
$P$ and $Q$ = constants

Using the above temperature reset scheme, the temperature will be increased up to the point at which the rate of change of expected cost with respect to temperature is at its maximum. In the physical analogy, this rate is referred to as the specific heat of the mass (see van Laarhoven & Aarts [1987], pages 41–49). A clear exposition of this variation in our cooling schedule and the mathematical justification for it is provided in Abramson, *et al*, [1993].

# 6 Computational Results

In this section we present results of our SA algorithm. The test problems are taken from data used in Hoffman & Padberg [1992]. Table 1 presents the results of using the lower bound pruning technique (in combination with the SA). Table 2 shows the results achieved using basic SA. Table 3 shows the results from incorporating the structural properties into the algorithm.

Each problem originally consists of a number of rows and columns. After pre-processing

it is possible to remove redundant rows and columns using a technique described in Hoffman & Padberg [1992]. In this manner the problem size is reduced. As the SA algorithm proceeds it attempts to reduce the problem size further by using the reduced cost values in conjuction with the LP lower bound,as described in Section 3. Figure 2 shows a plot of the reduced costs plus the lower bound versus the column number for Test Problem # 1. It shows that providing the SA can produce a reasonably tight upper bound, a large portion of the columns can be pruned through the technique described in Section 3.

In Table 1 we show the number of columns at the beginning of the SA run and the number remaining at the end. It can be seen that in many cases (except Problem # 8 and Problem # 9) if the LP lower bounds are continually used in conjunction with the SA upper bounds, the number of columns in the problem (and hence the search space) is reduced dramatically. Table 1 also presents the known optimal cost for each of the test problems, together with the best and average costs returned by the SA (from 6 independent runs). The average times are also presented.

| Test No | # of (R X C) | # of (R X C) After PreProcessing | # of C After SA | Known Opt Cost | SA Best Cost | SA Avg Cost | SA Avg Time (Secs) | Pre Processing (Secs) | LP Solve Time (Secs) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 19 X 770 | 19 X 639 | 19 | 1004 | 1004 | 1004 | 30.8 | 3.5 | 2.8 |
| 2 | 18 X 2540 | 18 X 2034 | 38 | 426 | 426 | 426 | 150.2 | 18.9 | 15.2 |
| 3 | 26 X 2653 | 26 X 1884 | 36 | 393 | 393 | 393 | 111.0 | 22.0 | 11.8 |
| 4 | 26 X 2662 | 26 X 1823 | 36 | 802 | 802 | 802 | 155.9 | 24.3 | 13.8 |
| 5 | 23 X 3068 | 23 X 2415 | 29 | 667 | 667 | 667 | 1404.4 | 26.2 | 16.9 |
| 6 | 40 X 2879 | 32 X 2134 | 21 | 1086 | 1086 | 1086 | 608.3 | 30.7 | 16.0 |
| 7* | 50 X 6774 | 38 X 5956 | 81 | 779 | 779 | 1244 | 13182.4 | 127.2 | 93.0 |
| 8 | 55 X 7479 | 47 X 5915 | 5915 | 106 | 116 | 121 | 7456.8 | 482.0 | 132.9 |
| 9 | 531 X 5198 | 360 X 3846 | 3846 | 3049 | 4797 | 5213 | 18801.2 | 656.7 | 783.9 |

R refers to the number of rows and C refers to the number of columns in the problem.

**Table 1: Results of using lower bound pruning technique**

In order to demonstrate that the performance of the basic SA algorithm is improved by using the problem reduction scheme described in Section 3, we ran the same problems without employing the problem reduction tests. Results of this experiment are provided

in Table 2. We report the final cost achieved after the same amount of time as when the pruning scheme was used, and also after a significantly larger time. They show that the pruning scheme consistently produces a higher quality solution in the same amount of time, and that after allocating significantly more time to the basic SA that the quality of the solution had not improved very much. This is due to the random nature of the basic SA algorithm and its inability to restrict itself to fruitful areas of search.

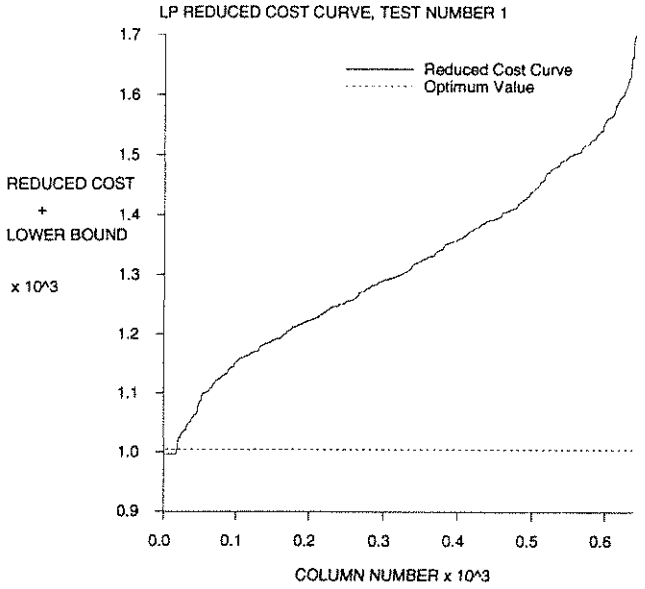


**Figure 2 Reduced costs for Problem #1**

| Test Number | # of (R X C) | SA Best Cost | SA Average Cost | SA Average Time (Secs) |
|---|---|---|---|---|
| 1 | 19 X 770 | 1021 | 1030 | 37.8 |
| | | 1004 | 1004 | 658.8 |
| 2 | 18 X 2540 | 474 | 504 | 200.3 |
| | | 433 | 444 | 3326.5 |
| 3 | 26 X 2653 | 442 | 597 | 98.6 |
| | | 393 | 450 | 3218.2 |
| 4 | 26 X 2662 | 1013 | 1051 | 150.4 |
| | | 802 | 804 | 1274.5 |
| 5 | 23 X 3068 | 867 | 938 | 773.9 |
| | | 718 | 802 | 5741.3 |
| 6 | 40 X 2879 | 1228 | 1298 | 641.7 |
| | | 1167 | 1188 | 6122.5 |

**Table 2: Results using normal SA without the lower bound pruning technique**

From Table 1, we see that in many cases the SA algorithm with pruning returned the known optimal solution at least once from the set of 6 runs, and in most cases it always returned the optimal solution. In a few of the runs for Problem 7 (marked with an asterisk in Table 1), however, the SA was not able to produce a sufficiently tight upper bound, and the number of columns could not be reduced during the run. These runs might be handled differently, by imposing an artificial upper bound before the SA started in order to exclude most of the columns from the search. Because the lower bound is sufficiently tight, we found that imposing an artificial upper bound of 10% higher than the lower bound always guaranteed that the solution lay in the neighbourhood of the optimal solution. Thus the SA always returned a feasible and optimal solution. However, the results in Table 1 do not incorporate this extension.

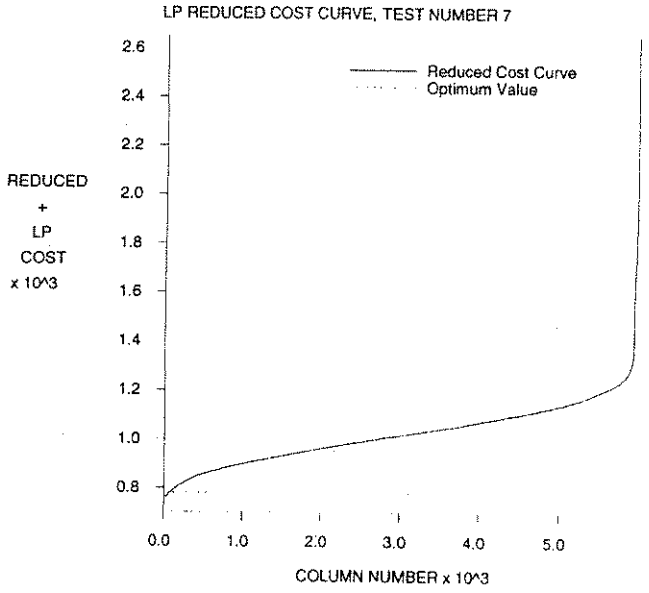


**Figure 3: Reduced costs for Problem #7**

The reason why the SA could not produce a sufficiently tight upper bound can be understood by considering the plot of the reduced cost against column number for Problem # 7 (shown in Figure 3) and comparing the shape with that for Problem # 1 (shown in Figure 2). It can be seen that there are many columns with very close

costs, all of which are potential candidates for inclusion in the final solution. Thus, the SA algorithm wanders through the search space swapping columns which are basically the same, and never gets a tight upper bound. By guessing at an intial upper bound which is close to the lower bound, it is possible to cut out many columns. The search can then proceed and find the optimal solution. If the guess had been too low then the set of columns excluded may have contained some which should be in the solution, and the SA procedure would have failed to produce a feasible solution.

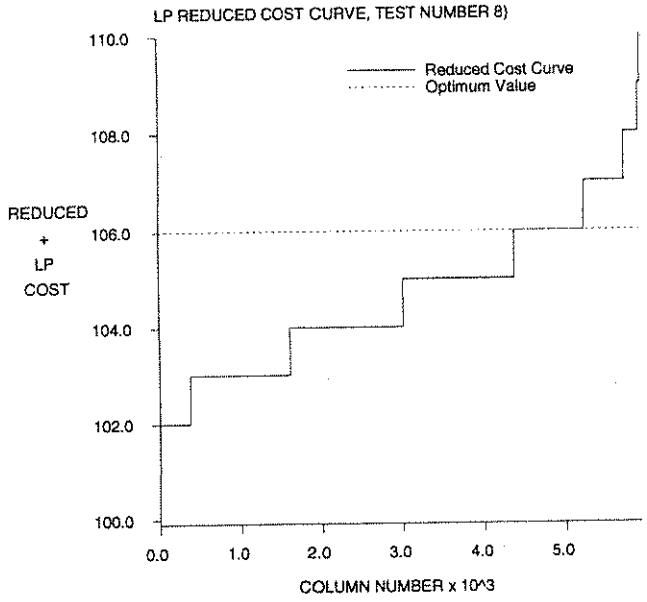


**Figure 4: Reduced costs for Problem #8**

Unfortunately, in the cases of Problem 8 and Problem 9, the method fails to prune the search space either by using the upper bound obtained during the SA run or by imposing an artificial upper bound. Consider the plot in Figure 4, Obviously the columns which have the reduced cost less than the optimum value will remain in the search space even if the optimal value is found. Hence even if the artificial upper bound is imposed, there still remains a large number of columns which prevent the SA from obtaining a tight upper bound. Also from limited experiments we have conducted, it appears as though Problem 9 is a special case in which there is only one feasible

solution. As the result of this, the final solution is infeasible. Hence, the addition of the set cost results in a local (and infeasible) solution, which is quite far from optimal.

Thus, we can conclude that SA with pruning is effective only when the relaxed LP produces a sufficiently tight lower bound and when the slope of the reduced cost curve is not too low. One would expect exactly the same behaviour from a branch and bound algorithm which relied on the same lower bound in order to prune nodes of the search tree.

Table 3 shows the effect of including the structural properties to constantly prune the search space within an SA algorithm. As in Table 1, the figures in Table 1 are averaged over 6 independent runs.

| Test Number | # of (R X C) | # of C After Annealing | Known Opitmum Cost | SA Best Cost | SA Average Cost | SA Average Time (Secs) |
|---|---|---|---|---|---|---|
| 1 | 19 X 770 | 11 | 1004 | 1004 | 1004 | 37.2 |
| 2 | 18 X 2540 | 38 | 426 | 426 | 426 | 212.2 |
| 3 | 26 X 2653 | 36 | 393 | 393 | 393 | 110.6 |
| 4 | 26 X 2662 | 36 | 802 | 802 | 802 | 166.0 |
| 5 | 23 X 3068 | 35 | 667 | 667 | 667 | 787.0 |
| 6 | 40 X 2879 | 21 | 1086 | 1086 | 1086 | 637.4 |

**Table 3: Using Structural properties to fix variables**

At first inspection, the results in Table 3 indicate that the use of the problems' structural properties does not improve the performance of our SA algorithm with search-space pruning based on LP lower bounds. In almost all cases the time to solution is actually slower than those reported in Table 1. However, the advantage of using structural properties to prune the search space is that, at the end of the SA run, we are often left with a very few variables in the problem. Hence, even an exhaustive search to prove optimality takes very little computational effort.

# 7 Conclusions

In this paper, we have presented a simulated annealing algorithm for the SPP, the underlying problem in airline crew scheduling problems. We have tested it on some practical–sized problems from the literature and the results indicate that the algorithm is quite powerful. Using clever pruning techniques, we are able to reduce the search space (by a large amount. The improved algorithm always out performs the basic simulated annealing algorithm, even when it cannot solve the problem). This method also enables us to comment on the quality of the final solution obtained. For some problems of a specific nature however, the technique does not find reasonable solutions. Further investigations need to be carried out to deal with these complications.

We have not fully investigated the scheme where only feasible moves are allowed in the algorithm. However, limited initial results seem to indicate that this scheme works better than the one in which infeasible moves are accepted with a resulting penalty. Under this scheme, the time to perform each move appears to be much longer than the current times. However, the overall time for the SA to complete appears to be quicker. Moreover, the quality of the final solution is also better. This particular aspect requires further investigation.

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